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Non-factorizable contributions in B decays revisited

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Abstract

$\bar{B} \rightarrow D\pi$, $D^*\pi$, $J/\psi\bar{K}$ and $J/\psi\pi$ decays are studied by decomposing their amplitude into a sum of factorizable and non-factorizable ones. The former is estimated by using the naive factorization while the latter is calculated by using a hard pion approximation in the infinite momentum frame.

Although experiments provide only upper bounds of the branching ratios for the color suppressed $\bar{B}^0 \rightarrow D^0\pi^0$ and $D^{*0}\pi^0$ decays, consistency among the branching ratios for the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays including the charged modes leads to phenomenologically allowed lower limits of $\mathcal{B}(\bar{B}^0 \rightarrow D^0\pi^0)$ and $\mathcal{B}(\bar{B}^0 \rightarrow D^{*0}\pi^0)$. Our result is compared with the phenomenologically estimated branching ratios as well as the measured ones. As the consequence, it is seen that the non-factorizable amplitude is rather small in the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays but can still efficiently interfere with the main (factorized) amplitude. In the color suppressed $\bar{B} \rightarrow J/\psi\bar{K}$ and $J/\psi\pi$ decays, non-factorizable contribution is more important. A sum of the factorized and non-factorizable amplitudes can improve the result from the factorization, although it is sensitive to model dependent values of form factors involved.

I. INTRODUCTION

The factorization (or vacuum insertion) prescription was first proposed long time ago [1] and, after the discovery of the standard model, it was revived [2], has been improved and applied extensively to hadronic weak decays of heavy mesons [3,4]. The improved one has been supported by two independent arguments, i.e., one is the large N_c (color degree of freedom) argument [5] that the factorizable amplitude which is given by the leading terms in the large N_c expansion dominates in hadronic weak decays and the other is the color transparency argument [6] that the factorization works well under a particular kinematical condition, i.e., a heavy quark decays into another heavy quark plus a pair of light quark and anti-quark which are emitted colinearly with sufficiently high energies, for example, as $b \rightarrow c + (\bar{u}d)_1$, where $(\bar{u}d)_1$ denotes a color singlet $(\bar{u}d)$ pair. In the former case, if the factorization works well in B decays, it will again work well in K and charm decays since the large N_c argument is independent of flavors. On the other hand, in the latter case, it cannot work well in K and charm decays where the kinematics cannot satisfy the above condition, even if it works well in B decays.

We here consider two body decays of charm mesons to check if the large N_c argument works well in hadronic weak interactions. A naive application of the factorization prescription to charm decays leads to the color suppression [suppression of color mismatched decays, $D^0 \rightarrow \bar{K}^0\pi^0$, $\bar{K}^{*0}\pi^0$, etc., described by $c \rightarrow (s\bar{d})_1 + u$]. It means, for example, that the amplitude for the D meson decays into isospin $I = \frac{1}{2}$ ($\bar{K}\pi$) final states is approximately cancelled by the one into $I = \frac{3}{2}$ ($\bar{K}\pi$) final states and hence the phases of these amplitudes are nearly equal to each other. Therefore the factorized amplitudes for two body decays of charm mesons should be approximately real except for the overall phase. However the measured rates for these decays are not compatible with the color suppression and the amplitudes for $D \rightarrow \bar{K}\pi$ and $\bar{K}^*\pi$ decays have large phase differences between the amplitudes for decays into the $I = \frac{1}{2}$ and $\frac{3}{2}$ final states [7]. To get rid of this problem, the factorization has been implemented by taking account for final state interactions. However, amplitudes with final state interactions are given by non-leading terms in the large N_c expansion as will be discussed later, so that the large N_c argument does not work well in charm decays and hence also in B decays since the large N_c argument is independent of flavors. On the other hand, if the color transparency argument works well, the factorization will be a good approximation in the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays while, in the $\bar{B} \rightarrow J/\psi\bar{K}$ and $J/\psi\pi$, the factorization cannot be a good approximation (in contrast with in the large N_c argument) but non-factorizable long distance contribution can play a role.

In this article, we study $\bar{B} \rightarrow D\pi$, $D^*\pi$, $J/\psi\bar{K}$ and $J/\psi\pi$ decays. In the next section, we will review briefly the effective weak Hamiltonian and our basic perspective. In Sec. III, the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays will be studied by decomposing their amplitude into a sum of factorizable and non-factorizable ones. The former will be estimated by using the naive factorization while the latter is calculated by using a hard pion approximation in the infinite momentum frame (IMF). We will estimate phenomenologically allowed branching ratios from the measured ones and compare our result with the estimated ones as well as the measured ones. As the consequence, it will be seen that the factorization works fairly well but non-factorizable contributions are still not negligible. In Sec. IV, the color suppressed decays, $\bar{B} \rightarrow J/\psi\bar{K}$ and $J/\psi\pi$, will be investigated in the same way. A brief summary will

be given in the final section.

II. EFFECTIVE WEAK HAMILTONIAN

Before we study amplitudes for B decays, we review briefly the $|\Delta B| = 1$ effective weak Hamiltonian. Its main part is usually written in the form

$$H_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{bc} \{c_1 O_1 + c_2 O_2\} + h.c., \quad (1)$$

where c_1 and c_2 are the Wilson coefficients of the four quark operators, O_1 and O_2 , respectively, given by products of color singlet left-handed currents,

$$O_1 =: (\bar{d}u)_{V-A}(\bar{c}b)_{V-A}: \quad \text{and} \quad O_2 =: (\bar{c}u)_{V-A}(\bar{d}b)_{V-A}:. \quad (2)$$

The renormalization scale (μ) dependence of c_i and O_i is not explicitly described unless it is required. V_{ij} 's denote the CKM matrix elements [8] which are taken to be real since CP invariance is always assumed in this article.

When we calculate factorizable amplitudes for the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays later, we use, as usual, the so-called BSW Hamiltonian [3,4]

$$H_w^{\text{BSW}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb} \{a_1 O_1 + a_2 O_2\} + h.c. \quad (3)$$

which can be obtained from Eq.(1) by using the Fierz reordering, where the operators O_1 and O_2 in Eq.(3) should be no longer Fierz reordered. The coefficients a_1 and a_2 are given by

$$a_1 = c_1 + \frac{c_2}{N_c}, \quad a_2 = c_2 + \frac{c_1}{N_c}, \quad (4)$$

where N_c is the color degree of freedom. When H_w^{BSW} is obtained, an extra term which is given by a color singlet sum of products of colored currents,

$$\tilde{H}_w = \frac{G_F}{\sqrt{2}} V_{ud} V_{cb} \{c_2 \tilde{O}_1 + c_1 \tilde{O}_2\} + h.c., \quad (5)$$

comes out, where

$$\tilde{O}_1 = 2 \sum_a : (\bar{d}t^a u)_{V-A}(\bar{c}t^a b)_{V-A}: \quad \text{and} \quad \tilde{O}_2 = 2 \sum_a : (\bar{c}t^a u)_{V-A}(\bar{d}t^a b)_{V-A}: \quad (6)$$

with the generators t^a of the color $SU_c(N_c)$ symmetry.

To realize physical amplitudes for B decays by matrix elements of \tilde{H}_w , soft gluon(s) should be exchanged between quarks which belong to different meson states or different currents. Therefore, amplitudes given by \tilde{H}_w correspond to non-leading terms in the large N_c expansion and are not factorizable, i.e., \tilde{H}_w is responsible for non-factorizable amplitudes. The above can be described schematically in Fig. 1, where we have not explicitly shown soft gluon exchange(s) between quarks belonging to the same meson and the same current for

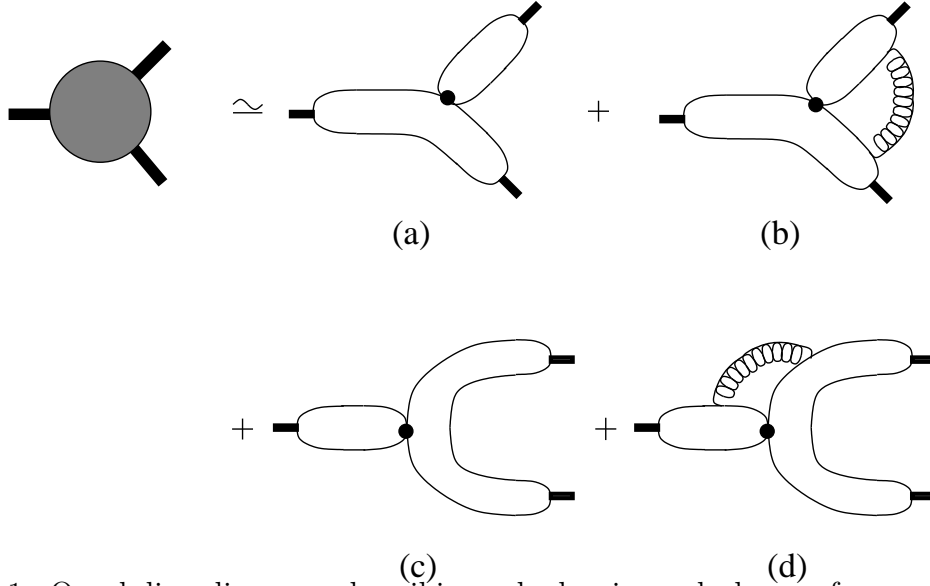


Fig. 1. Quark-line diagrams describing a hadronic weak decay of meson into two meson final states. Connectedness of quark-lines are assumed. The bullet denotes the weak vertex with perturbative QCD corrections and the spiral line denotes the soft-gluon(s). Soft-gluon exchange(s) between quarks belonging to the same meson and the same current are not explicitly described. The diagrams (a) and (c) correspond to the leading terms in the large N_c expansion while (b) and (d) to the non-leading terms.

simplicity. In the diagrams (a) and (c), no soft gluon(s) are exchanged between quarks belonging to different mesons or different currents. Therefore, the amplitudes described by these diagrams which correspond to the leading term in the large N_c expansion are factorizable. In these diagrams, the weak vertices are given by products of color singlet currents as in H_w^{BSW} . However, the diagrams (b) and (d) correspond to non-leading terms and are non-factorizable since soft gluon(s) are exchanged between quarks belonging to different mesons or different currents, so that the weak vertices in these diagrams are given by products of colored currents as in \tilde{H}_w . In the large N_c argument, the latter is neglected while, in the color transparency, it is not guaranteed that the latter can be neglected. Therefore, H_w^{BSW} and \tilde{H}_w are responsible for factorizable and non-factorizable amplitudes, respectively.

III. $\bar{B} \rightarrow D\pi$ AND $D^*\pi$ DECAYS

As discussed in the previous section, we start to study nonleptonic weak decays decomposing their amplitude into a sum of *factorizable* and *non-factorizable* ones [9,10]. The former is estimated by using the naive factorization in the BSW scheme [3]. The latter will be calculated later by using a hard pion approximation in the infinite momentum frame (IMF) [11,12]. The hard pion amplitude will be given by *asymptotic* matrix elements of \tilde{H}_w (matrix elements of \tilde{H}_w taken between single hadron states with infinite momentum).

Now we consider, as an example, the factorizable amplitude for the $B^-(p) \rightarrow D^0(p')\pi^-(q)$ decay. It is given by

$$\begin{aligned} M_{\text{FA}}(B^-(p) \rightarrow D^0(p')\pi^-(q)) \\ = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud} \{ a_1 \langle \pi^-(q) | (\bar{d}u)_{V-A} | 0 \rangle \langle D^0(p') | (\bar{c}b)_{V-A} | B^-(p) \rangle \\ + a_2 \langle D^0(p') | (\bar{c}u)_{V-A} | 0 \rangle \langle \pi^-(q) | (\bar{d}b)_{V-A} | B^-(p) \rangle \}, \end{aligned} \quad (7)$$

in the BSW scheme. Factorizable amplitudes for the other $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays also can be calculated in the same way. To evaluate these amplitudes, we use the following parameterization of matrix elements of currents [4],

$$\langle \pi(q) | A_\mu | 0 \rangle = -i f_\pi q_\mu, \quad (8)$$

$$\langle D(p') | V_\mu | \bar{B}(p) \rangle = \left\{ (p+p')_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right\} F_1^{(D\bar{B})}(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0^{(D\bar{B})}(q^2), \quad (9)$$

$$\begin{aligned} \langle D^*(p') | A_\mu | \bar{B}(p) \rangle = & \left\{ (m_B + m_{D^*}) \epsilon_\mu^*(p') A_1^{(D^*\bar{B})}(q^2) - \frac{\epsilon^*(p') \cdot q}{m_B + m_{D^*}} (p+p')_\mu A_2^{(D^*\bar{B})}(q^2) \right. \\ & \left. - 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_3^{(D^*\bar{B})}(q^2) \right\} + 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_0^{(D^*\bar{B})}(q^2), \end{aligned} \quad (10)$$

where $q = p - p'$ and the form factors satisfy

$$A_3^{(D^*\bar{B})}(q^2) = \frac{m_B + m_{D^*}}{2m_{D^*}} A_1^{(D^*\bar{B})}(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2^{(D^*\bar{B})}(q^2), \quad (11)$$

$$F_1^{(D\bar{B})}(0) = F_0^{(D\bar{B})}(0), \quad A_3^{(D^*\bar{B})}(0) = A_0^{(D^*\bar{B})}(0). \quad (12)$$

To get rid of useless imaginary unit in the amplitude, however, we take the following parameterization of matrix element of vector current [13],

$$\langle V(p') | V_\mu | 0 \rangle = -i f_V m_V \epsilon_\mu^*(p'), \quad (13)$$

which can be treated in parallel to those of axial vector currents in Eq.(8) in the IMF. Using these expressions of current matrix elements, we obtain the factorized amplitudes for the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays in Table 1, where we have put $m_\pi^2 = 0$ and factored out the CKM matrix elements. As seen in Table 1, the color suppressed amplitudes are proportional to the small coefficient, a_2 , and therefore they are suppressed.

Before we evaluate numerically the factorized amplitudes, we study non-factorizable amplitudes for the $\bar{B}(p) \rightarrow D(p')\pi(q)$ [and $D^*(p')\pi(q)$] decays. The amplitudes for hadronic weak (two-body) decays with final state interactions will be described by diagrams similar to (b) and (c) in Fig. 1 in which soft gluon(s) are replaced by $\{q\bar{q}\}$ pair(s) [i.e., meson(s)], so that such amplitudes will be given by hadron dynamics and correspond to non-leading terms in the large N_c expansion as mentioned before. Therefore, we assume that the non-factorizable amplitudes are dominated by dynamical contributions of various hadron states, and then calculate them using a hard pion technique with PCAC (partially conserved axial-vector

Table 1. Factorized amplitudes for $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays where $m_\pi^2 = 0$. The CKM matrix elements are factored out.

Decay	A_{FA}
$\bar{B}^0 \rightarrow D^+\pi^-$	$i\frac{G_F}{\sqrt{2}}a_1f_\pi(m_B^2 - m_D^2)F_0^{(D\bar{B})}(0)\left[1 - \left(\frac{a_2}{a_1}\right)\left(\frac{f_B}{f_\pi}\right)\left(\frac{m_D^2}{m_B^2 - m_D^2}\right)\frac{F_0^{(D\pi)}(m_B^2)}{F_0^{(D\bar{B})}(0)}\right]$
$\bar{B}^0 \rightarrow D^0\pi^0$	$-i\frac{G_F}{2}a_2f_Dm_B^2F_0^{(\pi\bar{B})}(m_D^2)\left[1 + \left(\frac{f_B}{f_D}\right)\left(\frac{m_D^2}{m_B^2}\right)\frac{F_0^{(D\pi)}(m_B^2)}{F_0^{(\pi\bar{B})}(m_D^2)}\right]$
$B^- \rightarrow D^0\pi^-$	$i\frac{G_F}{\sqrt{2}}a_1f_\pi(m_B^2 - m_D^2)F_0^{(D\bar{B})}(0)\left[1 + \left(\frac{a_2}{a_1}\right)\left(\frac{f_D}{f_\pi}\right)\left(\frac{m_B^2}{m_B^2 - m_D^2}\right)\frac{F_0^{(\pi\bar{B})}(m_D^2)}{F_0^{(D\bar{B})}(0)}\right]$
$\bar{B}^0 \rightarrow D^{*+}\pi^-$	$-i\frac{G_F}{\sqrt{2}}a_1f_\pi A_0^{(D^*\bar{B})}(0)\left[1 - \left(\frac{a_2}{a_1}\right)\left(\frac{f_B}{f_\pi}\right)\frac{A_0^{(D^*\pi)}(m_B^2)}{A_0^{(D^*\bar{B})}(0)}\right]2m_{D^*}\epsilon^*(p') \cdot p$
$\bar{B}^0 \rightarrow D^{*0}\pi^0$	$i\frac{G_F}{2}a_2f_{D^*}F_1^{(\pi\bar{B})}(m_{D^*}^2)\left[1 + \left(\frac{f_B}{f_{D^*}}\right)\frac{A_0^{(D^*\pi)}(m_B^2)}{F_1^{(\pi\bar{B})}(m_{D^*}^2)}\right]2m_{D^*}\epsilon^*(p') \cdot p$
$B^- \rightarrow D^{*0}\pi^-$	$-i\frac{G_F}{\sqrt{2}}a_1f_\pi A_0^{(D^*\bar{B})}(0)\left[1 + \left(\frac{a_2}{a_1}\right)\left(\frac{f_{D^*}}{f_\pi}\right)\frac{F_1^{(\pi\bar{B})}(m_{D^*}^2)}{A_0^{(D^*\bar{B})}(0)}\right]2m_{D^*}\epsilon^*(p') \cdot p$

current) in the IMF; i.e., $\mathbf{p} \rightarrow \infty$ [11,12],

$$M_{\text{NF}}(\bar{B} \rightarrow D^{[*]}\pi) \simeq \lim_{\mathbf{p} \rightarrow \infty, \mathbf{q} \rightarrow 0} M_{\text{NF}}(\bar{B} \rightarrow D^{[*]}\pi). \quad (14)$$

In this approximation, M_{NF} is given by a sum of equal-time commutator (ETC) term and surface term,

$$M_{\text{NF}}(\bar{B} \rightarrow D^{[*]}\pi) \simeq M_{\text{ETC}}(\bar{B} \rightarrow D^{[*]}\pi) + M_{\text{S}}(\bar{B} \rightarrow D^{[*]}\pi). \quad (15)$$

M_{ETC} has the same form as that in the old soft pion approximation [14]

$$M_{\text{ETC}}(\bar{B} \rightarrow D^{[*]}\pi) = \frac{i}{f_\pi} \langle D^{[*]} | [V_{\bar{\pi}}, \tilde{H}_w] | \bar{B} \rangle. \quad (16)$$

In the above equation, the commutation relation, $[V_\pi + A_\pi, \tilde{H}_w] = 0$, has been used, where V_π and A_π are isospin charge and its axial counterpart, respectively. The surface term, M_{S} , is given by

$$M_{\text{S}}(\bar{B} \rightarrow D^{[*]}\pi) = \lim_{\mathbf{p} \rightarrow \infty, \mathbf{q} \rightarrow 0} \left\{ -\frac{i}{f_\pi} q^\mu T_\mu^{[*]} \right\}, \quad (17)$$

with the hypothetical amplitude

$$T_\mu^{[*]} = i \int e^{iqx} \langle D^{[*]}(p') | T[A_\mu^{(\bar{\pi})} \tilde{H}_w] | \bar{B}(p) \rangle d^4x. \quad (18)$$

When a complete set of energy eigen states is inserted between products of axial vector current and \tilde{H}_w in $T_\mu^{[*]}$, contributions of single meson intermediate states survive in contrast with the soft pion approximation. In this way, M_S is provided by a sum of all possible pole amplitudes,

$$M_S = \sum_n M_S^{(n)} + \sum_l M_S^{(l)}. \quad (19)$$

$M_S^{(n)}$ and $M_S^{(l)}$ are pole amplitudes in the s - and u -channels, respectively, and given by

$$\begin{aligned} M_S^{(n)}(\bar{B} \rightarrow D^{[*]}\pi) &= -\frac{i}{f_\pi} \left(\frac{m_{D^{[*]}}^2 - m_B^2}{m_n^2 - m_B^2} \right) \langle D^{[*]} | A_\pi | n \rangle \langle n | \tilde{H}_w | \bar{B} \rangle, \\ M_S^{(l)}(\bar{B} \rightarrow D^{[*]}\pi) &= -\frac{i}{f_\pi} \left(\frac{m_{D^{[*]}}^2 - m_B^2}{m_l^2 - m_{D^{[*]}}^2} \right) \langle D^{[*]} | \tilde{H}_w | l \rangle \langle l | A_\pi | \bar{B} \rangle. \end{aligned} \quad (20)$$

Therefore, M_S provides corrections to the soft pion approximation, i.e., the present hard pion technique can be considered as an innovation of the old soft pion technique. (See, for more details and notations, Refs. [11] and [12].) n and l in Eq.(20) run over all possible single meson states, not only ordinary $\{q\bar{q}\}$, but also hybrid $\{q\bar{q}g\}$, four-quark $\{qq\bar{q}\bar{q}\}$, glue-balls, etc. However, $|n\rangle$ and $|l\rangle$ as well as the external states are energy eigen states in the present case. Since the states which sandwich \tilde{H}_w should conserve their spins in the rest frame and since Lorentz invariant amplitudes are considered, only the states $|n\rangle$ and $|l\rangle$ which conserve spins in the matrix elements, $\langle n | \tilde{H}_w | \bar{B} \rangle$ and $\langle D^{[*]} | \tilde{H}_w | l \rangle$, should be picked up [15]. Therefore, we discard vector meson pole contributions to the u -channel of pseudo-scalar(PS)-meson decays into two PS-meson states although we have taken such contributions in our previous papers [16]. Since the B meson mass m_B is much higher than those of charm mesons and since wave function overlappings between the ground-state $\{q\bar{q}\}_0$ and excited-state-meson states are expected to be small, however, excited meson contributions will be small in these decays and can be safely neglected. Therefore the hard pion amplitudes as the non-factorizable long distance ones are approximately described in terms of *asymptotic ground-state-meson matrix elements* (matrix elements taken between single ground-state-meson states with infinite momentum) of V_π , A_π and \tilde{H}_w .

Amplitudes for dynamical hadronic processes are usually decomposed into (*continuum contribution*) + (*Born term*). Since M_S has been given by a sum of pole amplitudes, M_{ETC} corresponds to the continuum contribution [17] which can develop a phase relative to the Born term. Therefore, we parameterize the ETC terms using isospin eigen amplitudes and their phases. Since the $D\pi$ final state can have isospin $I = \frac{1}{2}$ and $\frac{3}{2}$, $M_{\text{ETC}}(\bar{B} \rightarrow D\pi)$'s are parameterized as

$$M_{\text{ETC}}(\bar{B}^0 \rightarrow D^+ \pi^-) = \sqrt{\frac{1}{3}} M_{\text{ETC}}^{(3)} e^{i\tilde{\delta}_3} + \sqrt{\frac{2}{3}} M_{\text{ETC}}^{(1)} e^{i\tilde{\delta}_1}, \quad (21)$$

$$M_{\text{ETC}}(\bar{B}^0 \rightarrow D^0 \pi^0) = -\sqrt{\frac{2}{3}} M_{\text{ETC}}^{(3)} e^{i\tilde{\delta}_3} + \sqrt{\frac{1}{3}} M_{\text{ETC}}^{(1)} e^{i\tilde{\delta}_1}, \quad (22)$$

$$M_{\text{ETC}}(B^- \rightarrow D^0 \pi^-) = \sqrt{3} M_{\text{ETC}}^{(3)} e^{i\tilde{\delta}_3}, \quad (23)$$

where $M_{\text{ETC}}^{(2I)}$'s are the isospin eigen amplitudes with isospin I and $\tilde{\delta}_{2I}$'s are the corresponding phase shifts introduced. In the present approach, therefore, the final state interactions are included in the non-factorizable amplitudes. This is compatible with the fact that amplitudes with final state interactions are given by diagrams which belong to non-leading terms in the large N_c expansion.

Asymptotic matrix elements of V_π and A_π are parameterized as

$$\langle \pi^0 | V_{\pi^+} | \pi^- \rangle = \sqrt{2} \langle K^+ | V_{\pi^+} | K^0 \rangle = -\sqrt{2} \langle D^+ | V_{\pi^+} | D^0 \rangle = \sqrt{2} \langle B^+ | V_{\pi^+} | B^0 \rangle = \dots = \sqrt{2}, \quad (24)$$

$$\langle \rho^0 | A_{\pi^+} | \pi^- \rangle = \sqrt{2} \langle K^{*+} | A_{\pi^+} | K^0 \rangle = -\sqrt{2} \langle D^{*+} | A_{\pi^+} | D^0 \rangle = \sqrt{2} \langle B^{*+} | A_{\pi^+} | B^0 \rangle = \dots = h. \quad (25)$$

The above parameterization can be obtained by using asymptotic $SU_f(5)$ symmetry [18], or $SU_f(5)$ extension of the nonet symmetry in $SU_f(3)$. Asymptotic matrix elements of V_π between vector meson states can be obtained by exchanging PS-mesons for vector mesons with corresponding flavors in Eq.(24), for example, as $\pi^{0,-} \rightarrow \rho^{0,-}$, etc. The $SU_f(4)$ part of the above parameterization reproduces well [12,19] the observed values of decay rates, $\Gamma(D^* \rightarrow D\pi)$ and $\Gamma(D^* \rightarrow D\gamma)$.

In this way, we can describe the non-factorizable amplitudes for the $\bar{B} \rightarrow D\pi$ decays as

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^+ \pi^-) \simeq -i \frac{\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle}{f_\pi} \left\{ \left[\frac{4}{3} e^{i\tilde{\delta}_1} - \frac{1}{3} e^{i\tilde{\delta}_3} \right] + \dots \right\}, \quad (26)$$

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^0 \pi^0) \simeq -i \frac{\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle}{f_\pi} \left\{ \frac{\sqrt{2}}{3} \left[2e^{i\tilde{\delta}_1} + e^{i\tilde{\delta}_3} \right] + \dots \right\}, \quad (27)$$

$$M_{\text{NF}}(B^- \rightarrow D^0 \pi^-) \simeq i \frac{\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle}{f_\pi} \left\{ e^{i\tilde{\delta}_3} + \dots \right\}, \quad (28)$$

where the ellipses denote the neglected pole contributions.

In the case of the $\bar{B} \rightarrow D^* \pi$ decays, the matrix element $\langle V | \tilde{H}_w | P \rangle$ should vanish because of conservation of spin as discussed before, so that $M_{\text{ETC}}(\bar{B} \rightarrow D^* \pi)$ also should vanish but now D and B^* poles in the s - and u -channels, respectively, survive, i.e.,

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^{*+} \pi^-) \simeq \frac{i}{f_\pi} \langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_B^2 - m_D^2} \right) \sqrt{\frac{1}{2}} h + \dots, \quad (29)$$

$$M_{\text{NF}}(\bar{B}^0 \rightarrow D^{*0} \pi^0) \simeq \frac{i}{\sqrt{2} f_\pi} \left[\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_B^2 - m_D^2} \right) + \langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_{B^*}^2 - m_{D^*}^2} \right) \right] \sqrt{\frac{1}{2}} h + \dots, \quad (30)$$

$$M_{\text{NF}}(B^- \rightarrow D^{*0} \pi^-) \simeq -\frac{i}{f_\pi} \langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle \left(\frac{m_B^2 - m_{D^*}^2}{m_{B^*}^2 - m_{D^*}^2} \right) \sqrt{\frac{1}{2}} h + \dots, \quad (31)$$

where the ellipses denote the neglected excited meson contributions. Therefore the non-factorizable amplitudes in the hard pion approximation are controlled by the asymptotic ground-state-meson matrix elements of \tilde{H}_w (and the possible phases).

Now we evaluate the above amplitudes in the IMF. The factorized amplitudes in Table 1 contain many parameters which have not been measured by experiments, i.e., form factors,

$F_0^{(D\bar{B})}(q^2)$, $A_0^{(D^*\bar{B})}(q^2)$, $F_0^{(\pi\bar{B})}(q^2)$, $F_1^{(\pi\bar{B})}(q^2)$, etc., and decay constants, f_D , f_D^* , f_B , etc. The form factors $F_0^{(D\bar{B})}(0)$ and $A_0^{(D^*\bar{B})}(0)$ have been calculated by using the heavy quark effective theory (HQET) [20]. The other form factors are concerned with light mesons and therefore have to be estimated by some other models. In color favored decays, however, main parts of the factorized amplitudes depend on the form factor, $F_0^{(D\bar{B})}(0)$ or $A_0^{(D^*\bar{B})}(0)$, and the other form factors are included in minor terms proportional to a_2 . Therefore our result may not be lead to serious uncertainties although some model dependent values of the form factors are taken. (We will take, later, $F_0^{(D\bar{B})}(0) \simeq A_0^{(D^*\bar{B})}(0) \simeq 0.59$ as in Ref. [21], $F_0^{(\pi\bar{B})}(m_D^2) \simeq 0.30$ and $F_1^{(\pi\bar{B})}(m_{D^*}^2) \simeq 0.34$ as expected in pQCD [22].) In the color suppressed $\bar{B}^0 \rightarrow D^0\pi^0$ and $D^{*0}\pi^0$ decays, however, the factorized amplitudes are proportional to the form factors, $F_0^{(\pi\bar{B})}(m_D^2)$ and $F_1^{(\pi\bar{B})}(m_{D^*}^2)$, respectively. Since their values are model dependent, the result on the color suppressed decays may be a little ambiguous, if non-factorizable contribution is less important. For the decay constants of heavy mesons, we assume $f_D \simeq f_{D^*}$ (and $f_B \simeq f_{B^*}$) since D and D^* (B and B^*) are expected to be degenerate because of heavy quark symmetry [20] and are approximately degenerate in reality. Here we take $f_{D^*} \simeq f_D \simeq 211$ MeV and $f_{B^*} \simeq f_B \simeq 179$ MeV from a recent result of lattice QCD [23]. In this way, we can obtain the factorized amplitudes in the second column of Table 2, where we have neglected very small annihilation terms in the $\bar{B}^0 \rightarrow D^0\pi^0$ and $D^{*0}\pi^0$ decay amplitudes.

To evaluate the non-factorizable amplitudes, we need to know the size of the asymptotic matrix elements of \tilde{H}_w and A_π . The asymptotic matrix elements of A_π which was parameterized in Eq.(25) is estimated [11,12] to be $|h| \simeq 1.0$ by using PCAC and the observed rate [24], $\Gamma(\rho \rightarrow \pi\pi)_{\text{exp}} \simeq 150$ MeV. We here take its positive sign, i.e., $h \simeq 1.0$, since, in the IMF, $\langle \rho^0 | A_{\pi^+} | \pi^- \rangle$ is given by the form factor $A_3^{(\rho\pi)}(0)$ included in the matrix element of axial-vector current. In this way, the factorized and non-factorizable amplitudes for the $B^- \rightarrow D^{*0}\pi^-$ decay interfere constructively with each other. For the asymptotic matrix elements, $\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle$ and $\langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle$, we treat them as unknown parameters and search phenomenologically for their values to reproduce the observed rates for the $\bar{B} \rightarrow D^{[*]}\pi$ decays. To this, we assume

$$\langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle \simeq \langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle \quad (32)$$

as expected in the heavy quark symmetry and parameterize these matrix elements using factorizable ones of H_w^{BSW} as

$$\langle D^0 | \tilde{H}_w | \bar{B}^0 \rangle = B_H \langle D^0 | H_w^{\text{BSW}} | \bar{B}^0 \rangle_{\text{FA}} \quad (33)$$

with

$$\langle D^0 | H_w^{\text{BSW}} | \bar{B}^0 \rangle_{\text{FA}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud} \left(\frac{m_D^2 + m_B^2}{2} \right) f_D f_B a_2, \quad (34)$$

where B_H is a parameter introduced. In this way, we obtain the hard pion amplitudes as the non-factorizable contributions listed in the third column of Table 2, where the CKM matrix elements have been factored out.

Before we compare our result with the observations, we study phenomenologically allowed branching ratios for the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays. For the color suppressed $\bar{B}^0 \rightarrow D^0\pi^0$ and $D^{*0}\pi^0$ decays, in particular, only the upper limits of their branching ratios have been

Table 2. Factorized and non-factorizable amplitudes for the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays. The CKM matrix elements are factored out.

Decay	$A_{\text{FA}} (\times 10^{-5} \text{ GeV})$	$A_{\text{NF}} (\times 10^{-5} \text{ GeV})$
$\bar{B}^0 \rightarrow D^+\pi^-$	$1.57 a_1 \left\{ 1 - 0.03 \left(\frac{a_2}{a_1} \right) \right\}$	$-3.70 a_2 B_H \left\{ \left[\frac{4}{3} e^{i\tilde{\delta}_1} - \frac{1}{3} e^{i\tilde{\delta}_3} \right] \right\}$
$\bar{B}^0 \rightarrow D^0\pi^0$	$-1.03 a_2 \left\{ \frac{f_D}{0.211 \text{ GeV}} \right\}$	$-3.70 a_2 B_H \left\{ \frac{\sqrt{2}}{3} [2e^{i\tilde{\delta}_1} + e^{i\tilde{\delta}_3}] \right\}$
$B^- \rightarrow D^0\pi^-$	$1.57 a_1 \left\{ 1 + 0.94 \left(\frac{a_2}{a_1} \right) \right\}$	$3.70 a_2 B_H \left\{ e^{i\tilde{\delta}_3} \right\}$
$\bar{B}^0 \rightarrow D^{*+}\pi^-$	$-1.53 a_1 \left\{ 1 - 0.22 \left(\frac{a_2}{a_1} \right) \right\}$	$-3.70 a_2 B_H \left\{ 0.694 \right\}$
$\bar{B}^0 \rightarrow D^{*0}\pi^0$	$0.997 a_2 \left\{ \frac{f_{D^*}}{0.211 \text{ GeV}} \right\}$	$3.70 a_2 B_H \left\{ 0.983 \right\}$
$B^- \rightarrow D^{*0}\pi^-$	$-1.53 a_1 \left\{ 1 + 0.92 \left(\frac{a_2}{a_1} \right) \right\}$	$-3.70 a_2 B_H \left\{ 0.696 \right\}$

given as their measured values at the present stage. However, we can estimate their lower bounds compatible with the measured branching ratios for the charged modes. To this, we parameterize the amplitudes for these decays as

$$A(\bar{B}^0 \rightarrow D^{[*]+}\pi^-) = \sqrt{\frac{1}{3}} A_3^{[*]} e^{i\delta_3^{[*]}} + \sqrt{\frac{2}{3}} A_1^{[*]} e^{i\delta_1^{[*]}}, \quad (35)$$

$$A(\bar{B}^0 \rightarrow D^{[*]0}\pi^0) = -\sqrt{\frac{2}{3}} A_3^{[*]} e^{i\delta_3^{[*]}} + \sqrt{\frac{1}{3}} A_1^{[*]} e^{i\delta_1^{[*]}}, \quad (36)$$

$$A(B^- \rightarrow D^{[*]0}\pi^-) = \sqrt{3} A_3^{[*]} e^{i\delta_3^{[*]}}, \quad (37)$$

since the $D\pi$ [and $D^*\pi$] final states can have $I = \frac{1}{2}$ and $\frac{3}{2}$, where $A_{2I}^{[*]}$'s and $\delta_{2I}^{[*]}$'s are isospin eigen amplitudes for the $\bar{B} \rightarrow D^{[*]}\pi$ decays and their phases, respectively. Taking positive values of the ratio of isospin eigen amplitudes, $r^{[*]} = A_3^{[*]}/A_1^{[*]}$, we obtain

$$\cos(\delta^{[*]}) = \left(\frac{9R_{0-}^{[*]} - 1}{4} \right) r^{[*]} - \frac{1}{r^{[*]}}, \quad (38)$$

where

$$\delta^{[*]} = \delta_1^{[*]} - \delta_3^{[*]} \quad \text{and} \quad r^{[*]} = \sqrt{\frac{1}{3R_{00}^{[*]}R_{0-}^{[*]} + 3R_{0-}^{[*]} - 1}}. \quad (39)$$

Here $R_{0-}^{[*]}$ and $R_{00}^{[*]}$ are ratios of decay rates,

$$R_{0-}^{[*]} = \frac{\Gamma(\bar{B}^0 \rightarrow D^{[*]+}\pi^-)}{\Gamma(B^- \rightarrow D^{[*]0}\pi^-)} \quad \text{and} \quad R_{00}^{[*]} = \frac{\Gamma(\bar{B}^0 \rightarrow D^{[*]0}\pi^0)}{\Gamma(\bar{B}^0 \rightarrow D^{[*]+}\pi^-)}. \quad (40)$$

Table 3. Branching ratios (%) for $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays, where the values of the CKM matrix elements, $V_{cb} = 0.040$ and $V_{ud} = 0.97$, the lifetimes, $\tau(B^-) = 1.65 \times 10^{-12}$ s and $\tau(\bar{B}^0) = 1.55 \times 10^{-12}$ s from the updated experimental data [24], $a_1 = 1.024$ with the LO QCD corrections and phenomenological $a_2 = 0.21$, $B_H = 0.17$, $\tilde{\delta}_1 = 88^\circ$ and $\tilde{\delta}_3 = 8^\circ$ are taken. \mathcal{B}_{FA} and \mathcal{B}_{tot} are given by the factorized amplitude and a sum of the factorized and non-factorizable ones, respectively. The values of \mathcal{B}_{ph} are estimated in the text.

Decays	\mathcal{B}_{FA}	\mathcal{B}_{tot}	\mathcal{B}_{ph}	$\mathcal{B}_{\text{exp}} (*)$
$\bar{B}^0 \rightarrow D^+\pi^-$	0.29	0.30	$0.28 - 0.34$	0.30 ± 0.04
$\bar{B}^0 \rightarrow D^0\pi^0$	0.005	0.011	$0.006 - 0.012$	< 0.012
$B^- \rightarrow D^0\pi^-$	0.45	0.52	0.53 ± 0.05	0.53 ± 0.05
$\bar{B}^0 \rightarrow D^{*+}\pi^-$	0.25	0.28	0.276 ± 0.021	0.276 ± 0.021
$\bar{B}^0 \rightarrow D^{*0}\pi^0$	0.005	0.013	$0.004 - 0.044$	< 0.044
$B^- \rightarrow D^{*0}\pi^-$	0.42	0.46	0.46 ± 0.04	0.46 ± 0.04

(*) The data values are taken from Ref. [24].

Values of $R_{0-}^{[*]}$ and $R_{00}^{[*]}$ can be estimated phenomenologically from the experimental data [24] on branching ratios for $\bar{B} \rightarrow D^{[*]}\pi$ decays in Table 3 as $R_{0-} = 0.58 \pm 0.10$, $R_{00} < 0.04$ and $R_{0-}^* = 0.61 \pm 0.08$, $R_{00}^* < 0.16$. However, these values of R_{0-} and R_{00} [R_{0-}^* and R_{00}^*] are not always compatible with each other, i.e., the right-hand-side (r.h.s.) in Eq.(38) is not always less than unity. It is satisfied in more restricted regions of R and R^* , i.e., approximately, $0.04 \gtrsim R_{00} \gtrsim 0.02$ and $0.68 \gtrsim R_{0-} \gtrsim 0.61$ for the $\bar{B} \rightarrow D\pi$ decays, and $0.16 \gtrsim R_{00}^* \gtrsim 0.02$ and $0.69 \gtrsim R_{0-}^* \gtrsim 0.53$ for the $\bar{B} \rightarrow D^*\pi$ decays. These values of R and R^* lead approximately to the phenomenological branching ratios, \mathcal{B}_{ph} 's in Table 3, when the experimental data, $\mathcal{B}(B^- \rightarrow D^0\pi^-)_{\text{exp}} = 0.53 \pm 0.05$ and $\mathcal{B}(B^- \rightarrow D^{*0}\pi^-)_{\text{exp}} = 0.46 \pm 0.04$, are fixed. Here we put $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\pi^-)_{\text{ph}} = \mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\pi^-)_{\text{exp}}$ since $\cos(\delta^*) \leq 1$ is satisfied for all the experimentally allowed values of R_{0-}^* . The allowed values of $\cos(\delta)$ are limited within a narrow region, $0.96 \lesssim \cos(\delta) \leq 1$, in which $\cos(\delta)$ is very close to unity while $\cos(\delta^*)$ is a little more mildly restricted (at the present stage) compared with the above $\cos(\delta)$, i.e., approximately, $0.70 \lesssim \cos(\delta^*) \leq 1$. Therefore, the phase difference between the $I = \frac{1}{2}$ and $\frac{3}{2}$ amplitudes for the $\bar{B} \rightarrow D\pi$ decays is very small while, in the $\bar{B} \rightarrow D^*\pi$ decays, the allowed region of the corresponding phase difference is broader at the present stage.

We now compare our result on the branching ratios, $\mathcal{B}(\bar{B} \rightarrow D\pi)$ and $\mathcal{B}(\bar{B} \rightarrow D^*\pi)$, with their phenomenologically estimated values (and experimental data), taking a sum of the factorized amplitude and the non-factorizable amplitude as the total one. To this, we determine values of parameters involved. We take $V_{cb} = 0.040$ from the updated value $|V_{cb}| = 0.0402 \pm 0.0019$ [24]. For the coefficients a_1 and a_2 in H_w^{BSW} , we do not know their true values. It has been known [25] that the NLO corrections to a_1 are small while the corresponding ones to a_2 may be not much smaller compared with the LO corrections and depend strongly on the renormalization scheme. Therefore, we expect that the value,

$a_1 = 1.024$ with the LO corrections at the scale $\mu \simeq m_b$ [25], is not very far from the true value, and hence we take the above value of a_1 . For a_2 , however, we consider two cases. As the first case (i), we take $a_2 = 0.125$ with the LO QCD corrections at $\mu \simeq m_b$ [25], and then we treat it as an adjustable parameter around the above value of a_2 as the second case (ii). For the phases $\tilde{\delta}_1$ and $\tilde{\delta}_3$ arising from contributions of non-resonant multi-hadron intermediate states into isospin $I = \frac{1}{2}$ and $\frac{3}{2}$ final states, they are restricted in the region $|\tilde{\delta}_{2I}| < 90^\circ$ since resonant contributions have already been extracted as pole amplitudes in M_S although their contributions are neglected as discussed before. For B_H , we here treat it as a free parameter but expect to be less than unity.

We now search for values of parameters, $\tilde{\delta}_1$, $\tilde{\delta}_3$ and B_H in the case (i), and a_2 , $\tilde{\delta}_1$, $\tilde{\delta}_3$ and B_H in the case (ii), which reproduce the phenomenologically estimated branching ratios (from the observed ones) for the $\bar{B} \rightarrow D^{[*]}\pi$ decays. In the case (i), it is hard to reproduce them simultaneously. In the case (ii), however, large $\tilde{\delta}_1$, ($90^\circ > \tilde{\delta}_1 \gtrsim 60^\circ$), and small $|\tilde{\delta}_3|$ are favored (but our result is not very sensitive to the latter). For a_2 and B_H , larger values of a_2 , ($0.26 \gtrsim a_2 \gtrsim 0.16$), compared with $a_2 = 0.125$ with the LO QCD corrections and rather small values of B_H , ($0.07 \lesssim B_H \lesssim 0.28$), are favored. We list our results on the branching ratios for $a_1 = 1.024$, $a_2 = 0.21$, $\tilde{\delta}_1 = 88^\circ$, $\tilde{\delta}_3 = 8^\circ$ and $B_H = 0.17$ in Table 3, where we have used the values, $V_{cb} = 0.040$, $V_{ud} = 0.97$, $\tau(B^-) = 1.65 \times 10^{-12}$ s and $\tau(\bar{B}^0) = 1.55 \times 10^{-12}$ s from the updated experimental data [24]. \mathcal{B}_{FA} and \mathcal{B}_{tot} are given by the factorized amplitude and a sum of the factorized and non-factorizable ones, respectively. Values of \mathcal{B}_{ph} have been obtained phenomenologically from \mathcal{B}_{exp} before. \mathcal{B}_{FA} , in which the non-factorizable contributions are discarded, reproduces fairly well the existing data. However, if we add the non-factorizable contributions, we can improve the fit to the phenomenologically estimated \mathcal{B}_{ph} . It is seen that the non-factorizable contributions to the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays are rather small but still can interfere efficiently with the main amplitude given by the naive factorization.

IV. $\bar{B} \rightarrow J/\psi \bar{K}$ AND $J/\psi \pi$ DECAYS

Now we study CKM-angle favored $\bar{B} \rightarrow J/\psi \bar{K}$ and suppressed $\bar{B} \rightarrow J/\psi \pi$ decays in the same way as in the previous section. Both of them are color mismatched and their kinematical condition is much different from the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays at the level of underlying quarks, i.e., $b \rightarrow (c\bar{c})_1 + s$ (or d) in the former but $b \rightarrow c + (\bar{u}d)_1$ in the latter. Therefore, if the large N_c argument does not work well in hadronic weak decays as seen before, dominance of factorized amplitudes in these decays cannot be guaranteed and hence non-factorizable long distance contribution can play an important role.

The factorized amplitude for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays is given by

$$M_{\text{FA}}(\bar{B} \rightarrow J/\psi \bar{K}) = -iV_{cb}V_{cs} \left\{ \frac{G_F}{\sqrt{2}} a_2 f_\psi F_1^{(\bar{K}\bar{B})}(m_\psi^2) \right\} 2m_\psi \epsilon^*(p') \cdot p. \quad (41)$$

The value of the decay constant of J/ψ is estimated to be $f_\psi \simeq 406$ MeV from the observed rate [24] for the $J/\psi \rightarrow \ell^+ \ell^-$. The value of the CKM matrix element V_{cs} is given by

Table 4. Branching ratios (%) for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays, where the values of $F_1^{(\bar{K}\bar{B})}(m_\psi^2)$ from the models, BSW, CDDFGN and GKP, in Refs. [3], [26] and [27], respectively, are used. Values of the other parameters involved are the same as in Table 3, where $B'_H = B_H$ is assumed. The data values are taken from Ref. [24].

Models	BSW	CDDFGN	GKP
$F_1^{(\bar{K}\bar{B})}(m_\psi^2)$	0.565	0.726	0.837
$\mathcal{B}_{\text{FA}}(\bar{B} \rightarrow J/\psi \bar{K})$ (%)	0.048	0.079	0.10
$\mathcal{B}_{\text{tot}}(\bar{B} \rightarrow J/\psi \bar{K})$ (%)	0.075	0.11	0.14
$\mathcal{B}_{\text{tot}}(\bar{B} \rightarrow J/\psi \bar{K})$ (%)	0.075	0.11	0.14
Experiments	$\mathcal{B}(B^- \rightarrow J/\psi K^-) = (0.100 \pm 0.010) \%$ $\mathcal{B}(\bar{B}^0 \rightarrow J/\psi \bar{K}^0) = (0.089 \pm 0.012) \%$		

$V_{cs} \simeq V_{ud} \simeq 0.97$. The form factor $F_1^{(\bar{K}\bar{B})}(m_\psi^2)$ has not been measured and its theoretical estimates are model dependent. We pick out tentatively three typical values of $F_1^{(\bar{K}\bar{B})}(m_\psi^2)$ by the models, BSW [3], CDDFGN [26] and GKP [27], among many models and list the resulting $\mathcal{B}_{\text{FA}}(\bar{B} \rightarrow J/\psi \bar{K})$ in Table 4, where we have used the same values of parameters as before. \mathcal{B}_{FA} from the factorized amplitude for the value of $F_1^{(\bar{K}\bar{B})}(m_\psi^2)$ by BSW, which is close to the most recent value from pQCD [22] with $SU_f(3)$ symmetry, is about a half of the observations [24], $\mathcal{B}(B^- \rightarrow J/\psi K^-)_{\text{exp}} = (0.100 \pm 0.010) \%$ and $\mathcal{B}(\bar{B}^0 \rightarrow J/\psi \bar{K}^0)_{\text{exp}} = (0.089 \pm 0.012) \%$. It means that some other (non-factorizable) contribution is needed in this case. On the other hand, \mathcal{B}_{FA} 's for the higher values (CDDFGN and GKP) of the form factor saturate the above data values even if non-factorizable contributions are not included. It will be seen later that the predicted values of \mathcal{B}_{tot} including non-factorizable contributions is a little too large, in particular, in the case of GKP.

Non-factorizable contributions to these decays are estimated by using a hard kaon approximation which is a simple extension of the hard pion technique in the previous section. With this approximation and isospin symmetry, non-factorizable amplitude for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays is given by

$$M_{\text{NF}}(\bar{B} \rightarrow J/\psi \bar{K}) = \frac{i}{f_K} \langle \psi | H_w | \bar{B}_s^{*0} \rangle \left(\frac{m_B^2 - m_\psi^2}{m_{B_s^*}^2 - m_\psi^2} \right) \sqrt{\frac{1}{2}} h + \dots, \quad (42)$$

where the ellipsis denotes neglected contributions of excited mesons [28]. Here we have used $\langle \bar{B}_s^0 | V_{K^+} | B^- \rangle = -1$ and $\sqrt{2} \langle \bar{B}_s^{*0} | A_{K^+} | B^- \rangle = -h$ which are flavor $SU_f(3)$ extensions of Eqs.(24) and (25). Asymptotic matrix element, $\langle \psi | \tilde{H}_w | \bar{B}_s^{*0} \rangle$, is parameterized in a way similar to that of $\langle D^{*0} | \tilde{H}_w | \bar{B}^{*0} \rangle$, i.e., $\langle \psi | \tilde{H}_w | \bar{B}_s^{*0} \rangle = B'_H |\langle \psi | H_w^{\text{BSW}} | \bar{B}_s^{*0} \rangle_{\text{FA}}|$, where B'_H is a parameter corresponding to B_H in Eq. (33). Then the total amplitude for the $\bar{B} \rightarrow J/\psi \bar{K}$ decays is approximately given by

$$M_{\text{tot}}(\bar{B} \rightarrow J/\psi \bar{K}) \simeq -i V_{cb} V_{cs} \{ 6.12 F_1^{(\bar{K}\bar{B})}(m_\psi^2) + 5.27 B'_H \} a_2 \times 10^{-5} \text{ GeV}, \quad (43)$$

where $f_K \simeq 160$ MeV and $f_{B_s^*} \simeq f_{B_s} \simeq 204$ MeV from the updated lattice QCD result [23] have been taken.

When we take $a_2 = 0.21$ as before and assume tentatively $B'_H = B_H$ ($= 0.17$ as before), our result, \mathcal{B}_{tot} , for the smaller value of the form factor, $F_1^{(\bar{K}\bar{B})}(m_\psi^2)$, from BSW [or pQCD with $SU_f(3)$ symmetry] is improved considerably. Contrary, \mathcal{B}_{tot} for the larger values of the form factor, in particular, from GKP is beyond the measured ones as seen in Table 4.

For the CKM-angle suppressed $\bar{B} \rightarrow J/\psi\pi$, the same technique and values of parameters as the above lead to

$$\begin{aligned} M_{\text{tot}}(B^- \rightarrow J/\psi\pi^-) &\simeq -\sqrt{2}M_{\text{tot}}(\bar{B}^0 \rightarrow J/\psi\pi^0) \\ &\simeq -iV_{cb}V_{cd}\{6.12F_1^{(\pi\bar{B})}(m_\psi^2) + 6.64B'_H\}a_2 \times 10^{-5} \text{ GeV}. \end{aligned} \quad (44)$$

The first equation is consistent with the measurements, $\mathcal{B}(B^- \rightarrow J/\psi\pi^-)_{\text{exp}} = (5.1 \pm 1.5) \times 10^{-5}$ [24] and $\mathcal{B}(\bar{B}^0 \rightarrow J/\psi\pi^0) = (2.5_{-0.9}^{+1.1} \pm 0.2) \times 10^{-5}$ [29]. Using $F_1^{(\pi\bar{B})}(m_\psi^2) \simeq 0.56$ estimated by pQCD [22], which is close to the one of $F_1^{(\bar{K}\bar{B})}(m_\psi^2)$ from BSW, we obtain

$$\mathcal{B}_{\text{tot}}(B^- \rightarrow J/\psi\pi^-) \simeq 4.2 \times 10^{-5}, \quad (45)$$

which should be compared with the the above measurement.

As seen in Table 4, our result on the color suppressed $\bar{B} \rightarrow J/\psi\bar{K}$ and $\bar{B} \rightarrow J/\psi\pi$ decays is still sensitive to the values of both of $F_1^{(\pi\bar{B})}(m_\psi^2)$ [or $F_1^{(\bar{K}\bar{B})}(m_\psi^2)$] and B'_H . It implies that both of the factorized and non-factorizable amplitudes are important in these decays.

V. SUMMARY

In the previous sections, we have studied the $\bar{B} \rightarrow D\pi$, $D^*\pi$, $J/\psi\bar{K}$ and $J/\psi\pi$ decays describing their amplitude by a sum of factorizable and non-factorizable ones. The former has been estimated by using the naive factorization while the latter has been calculated by using a hard pion (or kaon) approximation in the infinite momentum frame. The so-called final state interactions (corresponding to non-leading terms in the large N_c expansion) have been included in the non-factorizable long distance contributions. The corresponding ones in the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays are rather small and therefore the final state interactions seem to be not very important in these decays although still not necessarily negligible. Next, we have investigated phenomenologically the $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays, and observed that the existing data on their branching ratios are not always compatible with each other, i.e., $\cos(\delta^{[*]})$ is over unity for some values of $R_{00}^{[*]}$ and $R_{0-}^{[*]}$, where $\delta^{[*]}$ denote phase differences between amplitudes for the decays into $I = \frac{1}{2}$ and $\frac{3}{2}$ $D^{[*]}\pi$ final states. Eliminating such values of $R_{00}^{[*]}$ and $R_{0-}^{[*]}$, we have obtained phenomenologically allowed values of branching ratios, \mathcal{B}_{ph} , in Table 3, which keep approximately $\cos(\delta^{[*]}) < 1$ and include lower limits on $\mathcal{B}(\bar{B} \rightarrow D^0\pi^0)$ and $\mathcal{B}(\bar{B} \rightarrow D^{*0}\pi^0)$.

By taking $a_1 \simeq 1.024$ with the LO QCD corrections and the phenomenological $a_2 \simeq 0.21$ which has been suggested previously [21,30], the observed branching ratios for these decays can be well reproduced in terms of a sum of the hard pion amplitude and the factorized one. Namely, the factorized amplitudes are dominant but not necessarily complete and

long distance hadron dynamics should be carefully taken into account in hadronic weak interactions of B mesons. In color suppressed $\bar{B}^0 \rightarrow D^0\pi^0$, $D^{*0}\pi^0$, $\bar{B} \rightarrow J/\psi\bar{K}$ and $J/\psi\pi^-$ decays, non-factorizable long distance contributions are more important. In particular, in the $\bar{B} \rightarrow J/\psi\bar{K}$ decays, long distance physics should be treated carefully. When $a_2 \simeq 0.125$ with the LO QCD corrections is taken instead of the phenomenological $a_2 \simeq 0.21$, it is hard to reproduce the measured $\mathcal{B}(\bar{B} \rightarrow J/\psi\bar{K})_{\text{exp}}$ and $\mathcal{B}(B^- \rightarrow J/\psi\pi^-)_{\text{exp}}$ even if a sum of factorized and non-factorizable amplitudes is taken as long as $B'_H = B_H \simeq 0.17$.

The non-factorizable amplitudes are proportional to asymptotic ground-state-meson matrix elements of \tilde{H}_w , i.e., B_H or B'_H . To reproduce the measured rates for the color favored $\bar{B} \rightarrow D\pi$ and $D^*\pi$ decays, the non-factorizable contributions may not be negligible ($B_H \neq 0$) while too large values of B_H and B'_H will lead to too large rates for the color suppressed decays. However, their numerical results are still ambiguous since the amplitudes for the color suppressed decays are sensitive to model dependent form factors.

Experimental data on exclusive decays, in particular, decays into charm-less final states, of B mesons are rapidly increasing after the B factories. In these decays, the factorization will not be a good approximation since the large N_c argument does not work in hadronic weak processes and since these decays are outside of applicability of the color transparency, so that non-factorizable contributions are expected to be very important in these decays. Besides, some of these decays are expected to play important roles in determination of CP -violating parameters. Therefore, non-factorizable contribution in exclusive decays of B mesons is now and will be, in near future, one of very important subjects.

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